BSc III Year

Paper Code: 395 (Applied Statistics)

Index Numbers

Unit 2

UNIFIED SYLLABUS OF STATISTICS
B.A. & B.Sc. Part- III

Paper II : Applied Statistics

UNIT – I

Time series – its different components, illustrations, additive and multiplicative models, determination of trend-graphic, semi-average, least square and moving average methods, measures of seasonal variation-simple average, ratio to moving average, ration to trend, link related method.

UNIT – II

Index number – its definition, application of index number, price relative and quantity or volume relatives, link and chain relative, problem involved in computation of index number, use of averages, simple aggregative and weighted average method. Laspeyre's, Paashe's and Fisher's index number, time and factor reversal tests of index numbers, consumer price index

UNIT – III

Demographic methods : Sources of demographic data – census, register, ad-hoc survey, hospital records, demographic profiles of Indian Censuses. Measurement of mortality, crude death rates, age specific death rates, infant mortality rates. Measurement of fertility – crude birth rate, general fertility rate, age-specific birth rate, total fertility rate, gross and net reproduction rate. Standardized death rates. Complete life table, its main features and construction (Abridged life table).

UNIT – IV

Control charts for variables and attributes. Sampling inspection by attributes – single and double sampling plans. Producer's and consumer's risk, OC, ASN, ATI functions AOQL and LTPD of sampling plans. Sampling inspection by variables – simple cases.

INDEX NUMBER

Index Number :- Index numbers are the indicators which reflect changes over a specified period of time in (i) prices of different commodities (ii) industerial production, (iii) sales (iv) imports and exports (v) cost of living etc. These are the numbers which express the value of a variable at any given date called the 'given period' as a percentage of the value of that variable at some standard date called the 'base period'. Thus index number is a statistical device which enables us to arrive at a single representative figure which gives the general level of price of the commodities in an extensive group.

Index Numbers as Economic Barometers :-

In the words of G. Simpson and Kafka, "Index numbers are today one of the most widely used statistical devices... They are used to take the pulse of the economy and they have come to be used as indicators of inflationary or deflationary tendencies". Just as in ~~Physics~~ Physics and Chemistry; barometer measures the atmospheric pressure or pressure of gases, so in Economics, index numbers measure the pressure of economic behaviour and are rightly termed as 'economic barometer' or 'barometer of economic activity'.

Problems in the construction of Index Number :-

No index number is an all-purpose Index number. Hence there are many problems involved in the construction of index numbers which are briefly discussed below:-

1. Purpose of Index Number
2. Selection of commodities.
3. Selection of Base Period. 2015
4. Collection of Data for Index Number
5. Type of Average to be used.
6. Selection of Appropriate weights.

1. Purpose of Index Number :- The purpose for which the index number is being constructed should be clearly and unambiguously stated. For instance if we want to construct an index number for measuring the change in the general price level, we have to take the wholesale prices of finished ~~goods~~ products, intermediate products, agricultural products, mineral products etc.

2. Selection of Commodities :- We should select the commodities which are relevent to the index no. The selection of commodities should by by judgement sampling, and not by random sampling. For example, if the purpose ~~to~~ of an index is to measure the cost of living of low income group (poor families), we should select only those commodities or items which are consumed by persons belonging to this group, and not to include the goods/services which are ordinarily consumed by middle-income or high income group.

3. **Selection of Base Period:-** All index numbers are constructed in reference to a period against which the comparisions are to be made such a reference period is known as 'base period' and the index for this period is always taken as 100. The following are the basic criteria for the choice of the base period.

(i) The 'base period' must be a normal period in the sense that it should be free from all sorts of abnormalities or chance fluctuations such as economic boom or depression, Labour strikes, wars, floods, earthquakes etc.

(ii) The 'base period' should not be too distant from the given period as the technology and circumstances change with lapse of time.

4. **Callection of Data :-** Data are to be callected on the items which are to be included in the construction of index numbers. The choice of items totally depend on the purpose of index number. The information-usually the prices, consumption or demand - should be collected carefully from the units selected in the sample.

5. **Type of Average to be used:-** An index number is a special type of average. Usually three types of averages are used given in order of their priority.

(i) Geometric Mean (GM)

(ii) Arithmetic Mean (A.M)

(iii) Median

(i) Geometric Mean is the most prefered one because :-

(a) it gives equal weights to equal ratios of change.

(b) the extreme values do not receive undue weights.

(c) geometric mean based indices are reversible.

(ii) Arithmetic mean is unduly affected by extreme values. Still it is widely used because of its easiness of computations.

(iii) Median, though easiest to calculate of all the three, completely ignores the extreme values. But it is seldom used.

6. **Selection of Appropriate Weights :-** There are two types of idices used in the construction of index numbers.

(i) Unweighted indices, in which no specific weights are assigned to various commodities.

(ii) Weighted indices, in which appropriate weights are assigned to various commodities.

Here it should be ~~no~~ strictly noted that all items do not carry the same importance with regard to their consumption or requirement. For instance, butter has less importance than milk, fruits have less importance than vegetables etc. Hence items are to be weighted according to their relative importance.

Notations

p_{ij} = price of jth commodity in the ith year

q_{ij} = quantity of jth commodity in the ith year

$v_{ij} = p_{ij} \times q_{ij}$, value of jth commodity in the ith year

where $j = 1, 2, ---, n$ and $i = 0, 1, 2, --- k$; 0 serving as the base year and 'i' as the given year.

The summation Σ will be taken over j from 1 to n. Thus we will write $\sum_{j=1}^{n} p_{ij} = \Sigma p_{ij}$, and $\sum_{j=1}^{n} q_{ij} = \Sigma q_{ij}$; which are the ith year price and quantity respectively.

In particular Σp_{oj} and Σq_{oj} refer to base year ~~year~~ price and base year quantity respectively.

Construction of Index Number :-

2015

Formula method for constructing price index number

(i) Simple (Unweighted) Aggregate Method :-

In this method, the total of the prices of commodities in given years is divided by the total of the prices of commodities in a base year and expressed as percentage. Thus price (or quantity) index for the ith year $(i = 1, 2, --- k)$ as compared to the base year $(i = 0)$ is given by

$$P_{oi} = \frac{\Sigma P_{ij}}{\Sigma P_{oj}} \times 100 \quad , \quad Q_{oi} = \frac{\Sigma q_{ij}}{\Sigma q_{oj}} \times 100 \quad \text{——— (1)}$$

Drawback of simple (Unweighted) Aggregate Method :-

(a) The price of various commodities may be in different units, e.g per litre, per meter, per ~~quit~~ quintal etc.

(b) The relative importance of various commodities is neglected.

2013

(ii) Weighted Aggregate Method :-

This method provides for the different commodities to exert their influence in the index number by assigning appropriate weights to each. If w_j (quantity consumed) is the weight associated with the jth commodity then the weighted aggregative price index is given by

$$P_{oi} = \frac{\Sigma P_{ij} w_j}{\Sigma P_{oj} w_j} \times 100 \quad \text{——— (2)}$$

By the use of different types of weights, a number of formulae such as Laspeyre's, ~~Pasch~~ Paasche's, Dorbish-Bowley, Marshal-Edgeworth, Irving Fisher's, Kelly's Prices Index have been emerged for the construction of index number.

2014

Laspeyre's Price Index Number (or Base Year Method):-

If we take $w_j = q_{oj}$ in ②, i.e. if the base year quantities are taken as weights then Laspeyr's aggregative price index number is given by

$$P_{oi}^{La} = \frac{\sum P_{ij} q_{oj}}{\sum P_{oj} q_{oj}} \times 100 \quad \text{--- ③}$$

The Laspeyre's index is also termed as L-formula.

2014

Paasche's Price Index Number (or Given Year Method):-

If we take $w_j = q_{ij}$ in ②, i.e if the current year quantities are taken as weights, then Paasches Price Index or P-formula is given by

$$P_{oi}^{Pa} = \frac{\sum P_{ij} q_{ij}}{\sum P_{oj} q_{ij}} \times 100 \quad \text{--- ④}$$

Dorbish-Bowley Price Index Number:-

This formula is the arithmetic mean of the Laspeyre's and Paasche's price indices and is given by

$$P_{oi}^{DB} = \frac{1}{2}\left[\frac{\sum P_{ij} q_{oj}}{\sum P_{oj} q_{oj}} + \frac{\sum P_{ij} q_{ij}}{\sum P_{ij} q_{ij}} \right] \times 100 \quad \text{--- ⑤}$$

i.e $\boxed{P_{oi}^{DB} = \frac{1}{2}\left[P_{oi}^{La} + P_{oi}^{Pa} \right]}$

2009

Marshal Edgeworth Price Index Number (or Bas and current Year Method):-

If we take $w_j = \frac{1}{2}(q_{oj} + q_{ij})^{1/2}$ in ②, i.e the arithmetic mean of base year and current year quantities, the Marshal Edgeworth (ME) formula for Price Index number is given by

$$P_{oi}^{ME} = \frac{\sum P_{ij}\left(\frac{q_{oj} + q_{ij}}{2}\right)}{\sum P_{oj}\left(\frac{q_{oj} + q_{ij}}{2}\right)} \times 100 \quad \text{--- ⑥}$$

Walsch Price Index Number:- If we take $w_j = (q_{oj} \times q_{ij})^{1/2}$ in ②, i.e geometrically crossed weighted aggregatives; Walsch Price Index Number is given by

$$P_{oi}^{Wa} = \frac{\sum P_{ij} \cdot (q_{oj} \cdot q_{ij})^{1/2}}{\sum P_{oj} (q_{oj} \cdot q_{ij})^{1/2}} \times 100 \quad \text{--- ⑦}$$

Irwing Fisher's 'Ideal' Index Number :-

Fisher's index is the geometric mean of Laspeyre's and Paasche's formula. Thus it is given by

$$P_{oi}^{F} = (P_{oi}^{La} \times P_{oi}^{Pa})^{1/2} = \left[\frac{\sum P_{ij} q_{oj}}{\sum P_{oj} q_{oj}} \cdot \frac{\sum P_{ij} q_{ij}}{\sum P_{oj} q_{ij}} \right]^{1/2} \times 100. \quad \text{—— (8)}$$

Kelly's Price Index (or Fixed weight Method) :-

If we take $w_j = q$ in (2) i.e instead of using base year or current year quantities as suggested by Laspeyre and Paasche, T.L. Kelly kept the quantity constant for all periods. Kelly's Price Index is given as

$$P_{oi}^{K} = \frac{\sum P_{ij} q}{\sum P_{oj} q} \times 100 \quad \text{—— (9)}$$

, where q is taken as AM or GM of the quantities of two, three or more years as weight.

Comparision of Laspeyre's and Paasche's index numbers :-

(i) In Laspeyre's index number, base year quantities are taken as weights and in Paasche's index number, the current year quantities are taken as weights.

From the practical point of view, Laspeyre's index is often preferred to Paasche's for the simple reason that Laspeyre's index weights are the base year quantities and do not change from the year to the next. On the other hand Paasche's index weights are the current year quantities and in most cases these weights are difficult to obtain.

2014.

(ii) Laspeyre's index number is said to have upward bias because it tends to overestimate the price rise, where as the Paasche's index number is said to have downward bias because it tends to under estimate the prise rise.

When the price increase, there is usually a reduction in the consumption of those items whose prices have increased. Hence using base year weights in the Laspeyre's index, we will be giving too much weight to the prices that have increased the most and the numerator will be too large. Due to similar considerations, Paasche's index number using given year weights underestimates the rise in price and hence has downward bias.

Remark → If changes in prices and quantities between the reference period and the base period are moderate, both Laspeyre's and Paasche's indices give nearly same values.

Quantity Index Numbers :-

(i) Laspeyre's Quantity Index Number; Base year prices are taken as weight.

Thus
$$Q_{oi}^{La} = \frac{\sum q_{ij} p_{oj}}{\sum q_{oj} p_{oj}} \times 100$$

(ii) Paasche's Quantity Index Number: current year prices are taken as weight.

Thus
$$Q_{oi}^{Pa} = \frac{\sum q_{ij} p_{ij}}{\sum q_{oj} p_{oj}} \times 100$$

(iii) Fisher's Index Number:
$$Q_{oi}^{F} = \sqrt{Q_{oi}^{La} \times Q_{oi}^{Pa}} = \sqrt{\frac{\sum q_{ij} p_{oj}}{\sum q_{oj} p_{oj}} \times \frac{\sum q_{ij} p_{ij}}{\sum q_{ij} p_{ij}}} \times 100.$$

* Quantity index numbers study the changes in the volume of goods produced, consumed or distributed like the indices of agricultural production, industrial production, imports and exports etc.

Value Index Numbers :-

Value index numbers are given by the aggregate expenditure for any given year expressed as a percentage of the same in the base year. Thus
$$V_{oi} = \frac{\sum p_{ij} q_{ij}}{\sum p_{oj} q_{oj}} \times 100$$

These indices are not as common as price and quantity indices.

2011, 2013

Errors in the Measurement of Price and Quantity Index Numbers and their control :-

If we are given the problem of measuring the price and quantity factors $P_{oi}(u)$ and $Q_{oi}(u)$ respectively of the value index $V_{oi}(u)$, they are subject to an error of measurement and this error has the following three components.

(i) Formula Error

(ii) Sampling Error

(iii) Homogeneity Error.

(i) Formula Error :- The formula error arises from the fact that there is no universally accepted formula that will measure the price change or quantity change of a given data. In such case for example, if we denote by D, the difference between P_{oi}^{La} and P_{oi}^{Pa} or between Q_{oi}^{La} and Q_{oi}^{Pa}, D provides a measure for the formula consistensy. Note that when D is negligible, the formula error tends to be absent.

Alternate Defn of Formula Error :-

The difference between the price (quantity) indices due to Laspeyre's and Paasche's is known as formula Error. If the difference between the two is negligible, the formula error tends to be absent. Let the difference be denoted by D, then-

$$D = \begin{cases} P_{oi}^{La} \sim P_{oi}^{Pa} & \text{; for Price index differences} \\ Q_{oi}^{La} \sim Q_{oi}^{Pa} & \text{; for quantity index differences.} \end{cases}$$

(ii) Sampling Error :- Index numbers are often constructed for n selected items out of total number of N items produced or used. Hence some errors may occur due to sampling of items. Thus if P_{oi}^{n} is the price based on n-sampled items and P_{oi}^{N} on total N items, (a similar statement about the quantity indices Q_{oi}^{n} and Q_{oi}^{N}) ; then the sampling Error (say S) is given by

$$S = \begin{cases} P_{oi}^{n} \sim P_{oi}^{N} & \text{, difference between price indices} \\ Q_{oi}^{n} \sim Q_{oi}^{N} & \text{, difference between quantity indices.} \end{cases}$$

Sampling Error can be reduced firstly by selecting a random sample of items. But in practice, a purposive sample is mostly taken including the items of vital importance and excluding the remaing ones.

(iii) Homogeneity Error :- The accuracy of an index number depends on the number of binary items. Hence, the error caused by consideration of only the binary items omitting the unique items present in the base and current periods is termed as homogeneity error. The extent of homogeneity error is measured through R-test defined as—

$$R = \frac{N_0 + N_i + N_{oi}}{N_0 + N_i} = \frac{\text{Unique Items}}{\text{Total Items.}} \qquad \text{——— (1)}$$

Where N_0 and N_i are the number of items in the base and current periods respectively and N_{oi} is the number of binary items.

The numerator in the formula ① gives the number of unique items and denominator is the total no. of items in the both the periods considered seperately. If there is no unique item, $N_o + N_i + N_{oi} = 0$ and hence $R = 0$. Again if there is no binary items, $N_{oi} = 0$ and hence $R = 1$. This shows that R lies between 0 and 1. $(0 \le R \le 1)$. A value of R equal to zero or near 0 is preferred for homogeneity error to be negligible, and this can be acheived by taking the base year not far from the current year.

Simple Average of Price Relatives :-

When this method is used to construct a price index number, first of all price relatives are obtained for the various items included in the index and then the average of these relatives is obtained using any one of the averages i.e arithmetic mean (AM) or geometric mean (GM). etc

(i) When AM is used, the formula for computing P_{oi} is —

$$P_{oi}^{AM} = \frac{1}{n} \sum \left(\frac{P_{ij}}{P_{oj}} \right) \times 100$$

(ii) When GM is used, the formula for computing P_{oi} is —

$$P_{oi}^{GM} = \left[\prod_{j=1}^{n} \left(\frac{P_{ij}}{P_{oj}} \right) \right]^{1/n} \times 100 = \text{Antilog} \left[2 + \frac{1}{n} \sum \log \left(\frac{P_{ij}}{P_{oj}} \right) \right]$$

Where n is the number of commodities and price relative $= \frac{P_{ij}}{P_{oj}} \times 100$

2013

Weighted Average of Relatives :-

If w_j is the weight assigned to jth commodity, then the general formulae of index numbers obtained on taking the weighted average of the price relatives become

$$P_{oi}(AM) = \frac{\sum w_j \left(\frac{P_{ij}}{P_{oj}} \right)}{\sum w_j} \times 100$$

and $$P_{oi}(GM) = \left[\prod_{j=1}^{n} \left(\frac{P_{ij}}{P_{oj}} \right)^{w_j} \right]^{1/\sum w_j} \times 100$$

$$\Rightarrow P_{oi}(GM) = \text{Antilog} \left[2 + \frac{1}{\sum w_j} \left\{ \sum w_j \log \left(\frac{P_{ij}}{P_{oj}} \right) \right\} \right]$$

2013

Weighted Average of Price Relative Index Number is same as that of Laspeyre's Index Number and Paasche's Index Number:—

Since weighted average of relatives is given by

$$P_{0i}\,(AM) = \frac{\sum \omega_j \left(\frac{P_{ij}}{P_{0j}}\right)}{\sum \omega_j} \times 100 \quad \text{---} \ (1)$$

in (1)

(i) If $\omega_j = P_{0j} q_{0j}$ i.e if base year values are taken as weights, then –

$$P_{0i}\,(AM) = \frac{\sum P_{0j} q_{0j} \cdot \left(\frac{P_{ij}}{P_{0j}}\right)}{\sum P_{0j} q_{0j}} \times 100 = \frac{\sum P_{ij} q_{0j}}{\sum P_{0j} q_{0j}} \times 100$$

$$\Rightarrow \boxed{P_{0i}(AM) = P_{0i}^{La}}$$

Thus weighted average of price relative index number is same as that of Laspeyre's index if $\omega_j = P_{0j} q_{0j}$.

2013

(ii) If $\omega_j = P_{0j} q_{ij}$ i.e if current year quantity corresponding to its base year prices are taken as base, then

$$P_{0i}\,(AM) = \frac{\sum P_{0j} q_{ij} \cdot \left(\frac{P_{ij}}{P_{0j}}\right)}{\sum P_{0j} q_{ij}} \times 100 = \frac{\sum P_{ij} q_{ij}}{\sum P_{0j} q_{ij}} \times 100$$

$$\Rightarrow \boxed{P_{0i}(AM) = P_{0i}^{Pa}}$$

Thus weighted average of price relative index number is same as that of Paasche's index no. if $\omega_j = P_{0j} q_{ij}$.

2013

Criteria of a Good Index Number (test of consistency or adequacy):—

Several formulae have been suggested for constructing index numbers and the problem is that of selecting most appropriate one. To overcome this problem, a number of mathematical test have been suggested. These tests are listed below.

(i) Unit test
(ii) Time Reversal test
(iii) Factor Reversal test
(iv) Circular test.

(i) Unit Test :— This test requires the formula for constructing the index numbers to be independent of units in which the prices and quantities of various commodities are quoted. This test is satisfied by all the index numbers except simple aggregative method.

(ii) Time Reversal Test :- This is one of the two most important tests proposed by Irving Fisher as a test of consistency for a good index number. According to this test, if the time script (say price) of any index formula be interchanged, then the resulting index should be the reciprocal of the original index. symbolically

$$P_{oi} = \frac{1}{P_{io}} \Rightarrow \boxed{P_{oi} \times P_{io} = 1}$$

Time Reversal Test for Laspeyre's index :-

We have $P_{oi}^{La} = \frac{\sum p_{ij} q_{oj}}{\sum p_{oj} q_{oj}}$ and $P_{io}^{La} = \frac{\sum p_{oj} q_{ij}}{\sum p_{ij} q_{ij}}$

$$\therefore P_{oi}^{La} \times P_{io}^{La} = \frac{\sum p_{ij} q_{oj}}{\sum p_{oj} q_{oj}} \times \frac{\sum p_{oj} q_{ij}}{\sum p_{ij} q_{ij}} \neq 1$$

Thus Laspeyre's index does not satisfy the time reversal test.

Time Reversal Test for Paasche's index :-

We have $P_{oi}^{Pa} = \frac{\sum p_{ij} q_{ij}}{\sum p_{oj} q_{ij}}$ and $P_{io}^{Pa} = \frac{\sum p_{oj} q_{oj}}{\sum p_{ij} q_{ij}}$

$$\therefore P_{oi}^{Pa} \times P_{io}^{Pa} = \frac{\sum p_{ij} q_{ij}}{\sum p_{oj} q_{ij}} \times \frac{\sum p_{oj} q_{oj}}{\sum p_{ij} q_{ij}} \neq 1$$

Thus Paasche's index also does not satisfy the time reversal test.

Time Reversal test for Fisher index :-

We have

$$P_{oi}^{F} = \left[\frac{\sum p_{ij} q_{oj}}{\sum p_{oj} q_{oj}} \times \frac{\sum p_{ij} q_{ij}}{\sum p_{oj} q_{ij}}\right]^{1/2}$$

and $$P_{io}^{F} = \left[\frac{\sum p_{oj} q_{ij}}{\sum p_{ij} q_{ij}} \times \frac{\sum p_{oj} q_{oj}}{\sum p_{ij} q_{oj}}\right]^{1/2}$$

$$\therefore P_{oi}^{F} \times P_{io}^{F} = \left[\frac{\sum p_{ij} q_{oj}}{\sum p_{oj} q_{oj}} \times \frac{\sum p_{ij} q_{ij}}{\sum p_{oj} q_{ij}}\right]^{1/2} \times \left[\frac{\sum p_{oj} q_{ij}}{\sum p_{ij} q_{ij}} \times \frac{\sum p_{oj} q_{oj}}{\sum p_{ij} q_{oj}}\right]^{1/2}$$

$$\Rightarrow P_{oi}^{F} \times P_{io}^{F} = 1$$

Hence Fisher's index number satisfies Time Reversal Test.

Hence Remark: The simple geometric mean of price relative and the Marshal Edgeworth formula (without factor 100) also satisfy time reversal test

(iii) Factor Revarsal Test :- This is the second most important test suggested by Irving Fisher. According to this test if P_{ij} and P_{oj} represent the prices ~~and~~ of current year and base year respectively, and q_{ij} and q_{oj} are the quantities of current year and base year respectively too. Also if P_{oi} and Q_{oi} are the price index no. and quantity index no. respectively then the factor reversal test is given by

$$P_{oi} \times Q_{oi} = V_{oi}$$

where V_{oi} is the value index number defined as

$$V_{oi} = \frac{\sum P_{ij} q_{ij}}{\sum P_{oj} q_{oj}} \times 100$$

Thus $$P_{oi} \times Q_{oi} = \frac{\sum P_{ij} q_{ij}}{\sum P_{oj} q_{oj}} \quad \{\text{without factor } 100\}.$$

Factor Reversal Test for Fisher Index :-

We have $$P^F_{oi} = \left[\frac{\sum P_{ij} q_{oj}}{\sum P_{oj} q_{oj}} \times \frac{\sum P_{ij} q_{ij}}{\sum P_{oj} \cdot q_{ij}}\right]^{1/2}$$

and $$Q^F_{oi} = \left[\frac{\sum q_{ij} P_{oj}}{\sum q_{oj} P_{oj}} \times \frac{\sum q_{ij} P_{ij}}{\sum q_{oj} P_{ij}}\right]^{1/2}$$

$$\therefore P^F_{oi} \times Q^F_{oi} = \left[\frac{\cancel{\sum P_{ij} q_{oj}}}{\sum P_{oj} q_{oj}} \times \frac{\sum P_{ij} q_{ij}}{\cancel{\sum P_{oj} q_{ij}}} \times \frac{\cancel{\sum q_{ij} P_{oj}}}{\sum q_{oj} P_{oj}} \times \frac{\sum q_{ij} P_{ij}}{\cancel{\sum q_{oj} P_{ij}}}\right]^{1/2}$$

$$\Rightarrow P^F_{oi} \times Q^F_{oi} = \frac{\sum P_{ij} q_{ij}}{\sum P_{oj} q_{oj}} = V_{oi}$$

Hence Fisher's index satisfies Factor Reversal Test. It may be pointed out that none of the other formulae satisfies the factor reversal test.

Remark → Since Fisher's index satisfies both Time Reversal and Factor Reversal Test, it is termed as ideal index number.

(iv) Circular Test :- This test is based on the shiftability of the base and is an extension of the time reversal test. The test is defined as

$$P_{ab} \times P_{bc} \times P_{ca} = 1 \quad ; \quad a \neq b \neq c$$

The simple geometric mean of price relative and the Kelly's fixed Weight method only satisfy this test.

Time and Factor Reversal Tests under the condition that Laspeyre's and Paasche's indices are same.

If Laspeyre's index is equal to the Paasche's index then by usual notation we have

$$P_{oi}^{La} = P_{oi}^{Pa} \Rightarrow \frac{\sum P_{ij} q_{oj}}{\sum P_{oj} q_{oj}} \times 100 = \frac{\sum P_{ij} q_{ij}}{\sum P_{oj} q_{ij}} \times 100$$

$$\Rightarrow \left(\sum P_{ij} q_{ij}\right)\left(\sum P_{oj} q_{ij}\right) = \left(\sum P_{ij} q_{ij}\right)\left(\sum P_{oj} q_{oj}\right) \quad \text{——— (1)}$$

If (1) holds, then it can be easily verified (without factor 100) that

(i) $P_{oi}^{La} \times P_{io}^{La} = 1$ and $P_{oi}^{Pa} \times P_{io}^{Pa} = 1$. ——— (2)

(ii) $P_{oi}^{La} \times Q_{oi}^{La} = V_{oi}$ and $P_{oi}^{Pa} \times Q_{oi}^{Pa} = V_{oi}$ ——— (3)

From eqn (2) and (3) we conclude that

If Laspeyre's price index and Paasche's price index are equal then both of these index numbers satisfies time reversal test and factor reversal test.

2009

Chain Index Numbers :- In fixed base method, the base remains same (constant) throughout the year. On the other hand in chain base method, the price relatives for each found from the prices of the immediately preceding year. Thus the base changes from year to year. Such index numbers are useful in comparing current year figures with the preceding year figures. The relatives which we found by this method are called link relatives. (LR).

(1) Thus LR for current year $= \dfrac{\text{Current year figure}}{\text{Previous year figure}} \times 100$

By using these link relatives (LR's), we can find the chain indices for each year using the formula given below.

(2) Chain index for current Year $= \dfrac{\text{LR of current Year} \times \text{CI of Previous Year}}{100}$.

Remark :- The fixed base index number computed from original data and chain index computed from link relatives gives the same value of the index provided that there is only one commodity whose indices are being constructed.

2009

Merits of Chain Index Numbers :-

(i) The chain base method is great significance in practice because in economic and business data, we are often concerned with making comparison with the previous period.

(ii) Chain base method does not require the recalculation if some more items are introduced or deleted from the old data.

(iii) Index numbers are calculated from the chain base method are free from seasonal and cyclical variations.

Demerits of Chain Index Numbers :-

(i) This method is not useful for long term comparison.

(ii) If there is any abnormal year in the series, it will effect the subsequent year also.

2009

Differences between Fixed Base and Chain Base Indices :-

S.No.	Chain Base	S.No.	Fixed Base
1.	Here the base year changes.	1.	Base year does not changes.
2.	Link relative method is used.	2.	No such link relative method is used.
3.	Its calculations are tedious.	3.	Its calculations are simple.
4.	It can not be computed if any one year is missing	4.	It can be computed if any year is missing
5.	It is suitable for short period	5.	It is suitable for long period.

Steps in the construction of Chain Indices :-

(i) Express the figures for each year as a percentage of the preceding year to obtain the link relatives (LR).

(ii) These Link Relatives are chained together by successive multiplication to to get chain indices (CI) by the following formula:

$$\text{Chain Index} = \frac{\text{LR of current year} \times \text{CI of previous year}}{100}.$$

Conversion of Chain Index Numbers to Fix Base Index :-

$$FBI = \frac{\text{Current year CBI} \times \text{Previous year CBI}}{100}$$

Classification of Index Numbers :- There are three types of Index Numbers.

(i) Price Index Numbers :- It measure the general changes in the retail or wholesale price level of a particular commodity or group of commodities.

(ii) Cost of Living Index Numbers :- These indices intended to study the effect of change in the price level on the cost of living of different classes of people.

(iii) Quantity Index Numbers :- These indices measure the change in quantity of goods manufactured in a factory eg. the indices of industrial production or agricultural production.

2014

Cost of Living Index Number (Consumer Price Index Number) :-

The cost of living index numbers measur the changes in level of prices of commodities which directly affects the cost of living of a specified group of persons at a specified place at different places. Different classes of people consume different types of commodities, consumption of items also vary from man to man, place to place and class to class of people. For example the cost of living of rickshaw puller at Varanasi is different from rikshawpuller at Kolkata. The consumer price index helps us in determining the effect of rise and fall in rise of different classes of consumers living in different geographical areas.

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Main Steps or Problems in Construction of Construction of Living Index :-

following are the main steps (problems) in constructing a cost of living index number :-

(1) Scope and Average :- First step is to specify the particular population group for whom the index is mean i.e whether it relates to labors, industrial workers, teachers, officers etc. Along with the class of people it is also necessary to decide the geographical area covered by the index such as city, or an industrial area etc.

(2) Family Budget Enquiry :- We select a sample of families from the class of people for whome the index is intended and scrutinize their budget in detail. The enquiry should be conducted during a normal period i.e a period free from economic booms

or depressions. The family budget enquiry gives information about the nature and quality of the commodities consumed by the people. The commodities are being classified under following heads.

(i) Food
(ii) Clothing
(iii) Fuel and Lighting
(iv) House rent
(v) Miscellaneous items.

(3) Retail Price of different Commodities:- The collection of retail prices is a very important and at the same time very difficult task because such prices may vary from place to place, shop to shop and person to person. Price quotation should be obtained from the local markets, where the class of people reside or from super bazaars or departmental stores from which they usually make their purchases.

Uses of Cost of Living Index Numbers:-

(1) Cost of living index numbers indicate whether the real wages are rising or falling. In other words, they are used for calculating the real wages and to determine the change in the purchasing power of money. We have:-

$$\text{Purchasing Power of Money} = \frac{1}{\text{Cost of Living Index Number}}$$

$$\text{Real wages} = \frac{\text{money wages}}{\text{Cost of Living Index Number}} \times 100$$

(2) Cost of living indices are used for the regulation of dearness allowance or the grant of ~~bonus~~ bonus to the workers so as to enable them to meet the increased cost of living

(3) These indices are also used for deflation of income and value series in national accounts.

(4) These indices are also used for analyzing markets for particular kinds of goods.

(5). Tax authorities need them to compute cost inflation indices to determine capital gains.

Methods for construction of cost of living index numbers :-

Cost of living index number is constructed by the following formulae

(1). Aggregate expenditure (or weighted aggregate) Method

(2) Family Budget Method or the method of weighted relatives

(1) Aggregate expenditure (or weighted aggregate) Method :-

In this method, the quanties of commodities consumed by the particular group in the base year are taken as weights. The formula is given by

$$\text{Consumer price index} = \frac{\sum p_{ij} q_{oj}}{\sum p_{oj} q_{oj}} \times 100 \quad \text{——} \quad (1)$$

The aggregate expenditure method is nothing but Laspeyre's index and is the most popular method of constructing cost of living index number.

(2) Family Budget Method (or weighted aggregate Method) :-

In this method, cost of living index is obtained on taking the weighted average of price relatives, the weights are the values of quantities consumed in the base year. Thus in usual notation, if we write -

$$\text{Price Relative } (P_j) = \frac{p_{ij}}{p_{oj}} \times 100 \quad \text{and} \quad w_j = p_{oj} q_{oj}, \quad j = 1, 2, \ldots n$$

$$\text{Then consumer price index} = \frac{\sum w_j P_j}{\sum w_j} \quad \text{——} \quad (2)$$

Remark It should be noted that the consumer price index numbers by both the methods agree, since

$$\frac{\sum w_j P_j}{\sum w_j} = \frac{\sum p_{oj} q_{oj} \left(\frac{p_{ij}}{p_{oj}}\right) \times 100}{\sum p_{oj} q_{oj}} = \frac{\sum p_{ij} q_{oj}}{\sum p_{oj} q_{oj}} \times 100$$

Which is same as the index, defined in (1).

Example 3.7

With usual notations, Laspeyre's and Paasche's index numbers are given as

$$P_{oj}^{La} = \frac{\sum p_{ij} q_{oj}}{\sum p_{oj} q_{oj}} \times 100 \quad \text{and} \quad P_{oj}^{Pa} = \frac{\sum p_{ij} q_{ij}}{\sum p_{oj} q_{ij}} \times 100$$

Since we are given that –

(i) $\frac{p_{ij}}{p_{oj}}$ = constant, (say K) $\Rightarrow p_{ij} = K p_{oj}$ —— ①

(ii) $\frac{q_{ij}}{q_{oj}}$ = constan, (say m) $\Rightarrow q_{ij} = m q_{oj}$ —— ②

Thus using result from ① in Laspeyre's formula we have

$$P_{oj}^{La} = \frac{K \sum p_{oj} q_{oj}}{\sum p_{oj} q_{oj}} \times 100 = 100K$$

Also using result from ① in Paasche's formula we have

$$P_{oj}^{Pa} = \frac{K \sum p_{oj} q_{ij}}{\sum p_{oj} q_{ij}} \times 100 = 100K$$

Hence we conclude that $P_{oj}^{La} = P_{oj}^{Pa}$.

Similarly of ② holds, then –

$$P_{oj}^{La} = \frac{\sum p_{ij} \left(\frac{q_{ij}}{m}\right)}{\sum p_{oj} \left(\frac{q_{ij}}{m}\right)} \times 100 = \frac{\sum p_{ij} q_{ij}}{\sum p_{oj} q_{ij}} \times 100 = P_{oj}^{Pa}.$$

Hence proved.

Example 3.8

Here $L(\cdot)$ denotes the Laspeyre's index and $P(\cdot)$ the Paasche's index. Thus by usual notations $L(\cdot)$ and $P(\cdot)$ are given as follows.

$$L(p) = \frac{\sum p_{ij} q_{oj}}{\sum p_{oj} q_{oj}} \times 100, \quad L(q) = \frac{\sum p_{oj} q_{ij}}{\sum p_{oj} q_{oj}} \times 100$$

Similarly

$$P(p) = \frac{\sum p_{ij} q_{ij}}{\sum p_{oj} q_{ij}} \times 100, \quad P(q) = \frac{\sum p_{ij} q_{ij}}{\sum p_{ij} q_{oj}} \times 100$$

$$\text{Now } L(p) \times P(q) = \frac{\cancel{\sum p_{ij} q_{oj}}}{\sum p_{oj} q_{oj}} \times 100 \times \frac{\sum p_{ij} q_{ij}}{\cancel{\sum p_{ij} q_{oj}}} \times 100 = \frac{\sum p_{ij} q_{ij}}{\sum p_{oj} q_{oj}} \times 100^2 \quad \text{—— ①}$$

Similarly we obtain $L(q) \times P(p) = \frac{\sum p_{ij} q_{ij}}{\sum p_{oj} q_{oj}} \times 100^2$ —— ②

from ① and ②, we have —

$$L(p) \times P(q) = L(q) \times P(p)$$

$$\Rightarrow \boxed{\frac{L(p)}{L(q)} = \frac{P(p)}{P(q)}}$$ Hence proved

Example 3.9

Since r_{xy} is the weighted correlation coefficient between x and Y; σ_x and σ_Y being the weighted SD of X and Y respectively. Then by defn of correlation between X and Y, we have

$$r_{XY} = \frac{cov(X, Y)}{\sigma_X \cdot \sigma_Y}$$

$$\Rightarrow r_{XY} \cdot \sigma_X \sigma_Y = cov(X, Y) = \frac{\sum \omega_j X_j Y_j}{\sum \omega_j} - \left(\frac{\sum \omega_j X_j}{\sum \omega_j}\right) \cdot \left(\frac{\sum \omega_j Y_j}{\sum \omega_j}\right) \quad \text{——— ①}$$

Where the summation over j from 1 to n.

We are given that

$$X_j = \frac{p_{ij}}{p_{oj}}, \quad Y_j = \frac{q_{ij}}{q_{oj}}, \quad \omega_j = p_{oj} q_{oj} \text{ and } V_{oi} = \frac{\sum p_{ij} q_{ij}}{\sum p_{oj} q_{oj}}$$

Substituting all corresponding values in ①, we have

$$r_{XY} \sigma_X \sigma_Y = \frac{\sum \cancel{p_{oj}} \cancel{q_{oj}} \cdot \frac{p_{ij}}{\cancel{p_{oj}}} \cdot \frac{q_{ij}}{\cancel{q_{oj}}}}{\sum p_{oj} q_{oj}} - \frac{\sum \cancel{p_{oj}} q_{oj} \cdot \frac{p_{ij}}{\cancel{p_{oj}}}}{\sum p_{oj} q_{oj}} \cdot \frac{\sum p_{oj} \cancel{q_{oj}} \frac{q_{ij}}{\cancel{q_{oj}}}}{\sum p_{oj} q_{oj}}$$

$$= \frac{\sum p_{ij} q_{ij}}{\sum p_{oj} q_{oj}} - \frac{\sum p_{ij} q_{oj}}{\sum p_{oj} q_{oj}} \cdot \frac{\sum p_{oj} q_{ij}}{\sum p_{oj} q_{oj}}$$

$$= \frac{\sum p_{ij} q_{ij}}{\sum p_{oj} q_{oj}} \left[1 - \frac{\sum p_{ij} q_{oj}}{\sum p_{oj} q_{oj}} \cdot \frac{\sum p_{oj} q_{ij}}{\sum p_{ij} q_{ij}}\right]$$

$$= V_{oi} \left[1 - P_{oi}^{La} \cdot (P_{oi}^{Pa})^{-1}\right]$$

$$\Rightarrow r_{XY} \sigma_X \sigma_Y = V_{oi} \left[1 - \frac{P_{oi}^{La}}{P_{oi}^{Pa}}\right]$$

$$\Rightarrow \boxed{\frac{P_{oi}^{La}}{P_{oi}^{Pa}} = 1 - \frac{r_{XY} \sigma_X \sigma_Y}{V_{oi}}} \quad \text{——— ②}$$

Hence proved

Remark (1) In practice, under normal economic conditions, we have

$$-1 < r_{xy} < 0 \text{ and consequently } P_{0i}^{La} > P_{0i}^{Pa}.$$

Remark (2) If $r_{xy} > 0$, then from ②, we get

$$\frac{P_{0i}^{La}}{P_{0i}^{Pa}} < 1 \Rightarrow P_{0i}^{La} < P_{0i}^{Pa}$$

and if $r_{xy} < 0$, then $\frac{P_{0i}^{L}}{P_{0i}^{Pa}} > 1 \Rightarrow P_{0i}^{La} > P_{0i}^{Pa}$

Hence if the correlation between the price relatives X and quantity relatives Y is positive (negative), then Laspeyre's index is less (greater) than Paasche's index.

Remark (3) If either $r_{xy} = 0$ or $\sigma_x = 0$ or $\sigma_y = 0$, then

$$\frac{P_{0i}^{La}}{P_{0i}^{Pa}} = 1 \Rightarrow P_{0i}^{La} = P_{0i}^{Pa}$$

~~Hence if either~~

Example 3.10 Prove that Fisher's ideal index number lies between Laspeyre's and Paasche's index numbers.

Solution Let 'L' denote the Laspeyre's index number and 'P' denote the Paasche's index number. Then there may be two situations –

(i) $L \leq P$ or (ii) $L \geq P$

We know that Fisher's index $= \sqrt{L \cdot P}$

Now consider $L \leq P$

$\Rightarrow L^2 \leq LP \quad \{\because L > 0$

$\Rightarrow L \leq \sqrt{LP}$ —— ①

Also $L \leq P$

$\Rightarrow LP \leq P^2 \quad \{\because P > 0$

$\Rightarrow \sqrt{LP} \leq P$ —— ②

From ① and ②, we have

$$\boxed{L \leq \sqrt{LP} \leq P}$$

Similarly it follows from the situation when $L \geq P$, we get

$$\boxed{L \geq \sqrt{LP} \geq P}$$

In particular if $L = P$, then Laspeyre's, Paasche's and Fisher's indices are all equal.

Fisher's index number is called ideal index number :-

The Fisher's index number is called ideal index number due to the following characteristics.

(1) It is based on the Geometric Mean which is theoretically considered as the best average of constructing index numbers.

(2) It takes into account both current and base year prices as quantities.

(3) It satisfies both time reversal and factor reversal test which are suggested by Fisher.

(4). The upward bias of Laspeyre's index and downward bias of Paasche's index are balanced to a great extent.

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Example 3.11

Show that Marshal Edgeworth Index number lies between Laspeyre's and Paasche's index numbers. More specifically

(a) If $P_{0i}^{La} < P_{0i}^{Pa}$ then $P_{0i}^{La} < P_{0i}^{ME} < P_{0i}^{Pa}$

(b) If $P_{0i}^{Pa} < P_{0i}^{La}$ then $P_{0i}^{Pa} < P_{0i}^{ME} < P_{0i}^{La}$

Solution

To establish this result we shall first prove the following lemma

Lema If a, b, c and d are positive numbers, then

$$\frac{a}{b} < \frac{c}{d} \Rightarrow \frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d} \quad \text{———} \; (*)$$

Proof If $\frac{a}{b} < \frac{c}{d}$ the $ad < bc$ ——— (1)

Adding ab to both sides of (1), we get

$$ad + ab < ab + bc$$

$$\Rightarrow a(b+d) < b(a+c)$$

$$\Rightarrow \frac{a}{b} < \frac{a+c}{b+d} \quad \text{———} \; (2)$$

Now adding cd to both sides of (1), we get

$$cd + ad < bc + cd$$

$$\Rightarrow d(a+c) < c(b+d)$$

$$\Rightarrow \frac{a+c}{b+d} < \frac{c}{d} \quad \text{——— (3)}$$

Combining (2) and (3), we get the require lemma as in (*),

$$\boxed{\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}} \quad \text{——— (*)}$$

(a) Now if $P_{0i}^{La} < P_{0i}^{Pa}$ then $\dfrac{\sum p_{ij} q_{0j}}{\sum p_{0j} q_{0j}} < \dfrac{\sum p_{ij} q_{ij}}{\sum p_{0j} q_{ij}}$

Using (*), we can write it as

$$\frac{\sum p_{ij} q_{0j}}{\sum p_{0j} q_{0j}} < \frac{\sum p_{ij} q_{0j} + \sum p_{ij} q_{ij}}{\sum p_{0j} q_{0j} + \sum p_{0j} q_{ij}} < \frac{\sum p_{ij} q_{ij}}{\sum p_{0j} q_{ij}}$$

$$\Rightarrow \boxed{P_{0i}^{La} < P_{0i}^{ME} < P_{0i}^{Pa}}$$

Hence proved.

(b) Similarly if $P_{0i}^{Pa} < P_{0i}^{La}$, then $\dfrac{\sum p_{ij} q_{ij}}{\sum p_{0j} q_{ij}} < \dfrac{\sum p_{ij} q_{0j}}{\sum p_{0j} q_{0j}}$

$$\Rightarrow \boxed{P_{0i}^{Pa} < P_{0i}^{ME} < P_{0i}^{La}}$$

Hence from (a) and (b), we conclude that Marshal Edgeworth index lies between Laspeyre's and Paasche's index numbers.

Q1 Prove that AM of Laspeyre's index number and Paasche's index number is greater than or equal to Fisher's index number.

Q2 Show that if $A(p) = \frac{1}{2}[L(p) + P(q)]$ then $A(p)$ is greater than the Fisher's index number.